Moving loads on railway bridges. The Spanish Code approach

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ABSTRACT: This paper presents the results of part of the research carried out by a committee in charge of the elaboration of the new Spanish Code of Actions in Railway Bridges. Following the work developed by the European Rail Research Institute (ERRI), the dynamic effects caused by the Spanish high-speed train TALGO have been studied and compared with other European trains. A simplified envelope of the impact coefficient is also presented. Finally, the train-bridge interaction has been analysed and the results compared with those obtained from simple models based on moving loads.

1 INTRODUCTION

The computation of the so-called impact coefficient on railway bridges has a long tradition since the first accidents occurred in metal bridges during the past century.

The coefficient depending on the bridge length was established as a simple means of weighting the static deflection to obtain an envelope of the dynamic effects (Waddell 1897) although several attempts to establish a formula from first principles were tried from the very beginning (Stokes 1847, Willis 1849, Bresse 1880, Bleich 1924, Inglis 1934).

The advent of the computer opened the possibility of treating more complicated problems (Biggs 1964, Fryba 1972) and to compare them with experiments developed by UIC under the heading of Question D-23 (ORE 1970). The matter seemed so settled that a formula was provided and an effort was launched to move to more complicated matters: limitation of bridge deflections in relation with the passenger comfort (ORE 1983).

A further impulse came with the fast trains under the limelight and several resonance effects were observed that affected specially the structural behaviour of the ballast due to high accelerations.

Recently the ERRI has developed different procedures to represent in a simplified manner the dynamic effects although some more work is needed yet.

In Spain the first compulsory impact formula was proposed by the Ministry of Public Works in 1925. In 1975, under the influence of UIC studies, a new one was proposed in the framework of a Code of Actions for Railway Bridges (MOPU 1975). Recently the Ministry of Public Works —currently Ministerio de Fomento— is engaged in the publication of a new Code of Actions so the matter of a formula embracing the resonant effects induced by the high-speed trains has been tackled.

In particular the authors have been scrutinizing and repeating some of the works developed by ERRI; this work is necessary because the Spanish high-speed lines can be used not only by the several European trains (TGV, Eurostar and ICE among others) but also by the Spanish TALGO train. In this paper we are going to show how the results compare with the ERRI ones when the aforementioned Talgo train is included.

2 NUMERICAL MODELLING

In order to compute the deflections and accelerations several computer programs have been developed ranging from the simplest one, dedicated to simply supported beams crossed by a series of loads, to the most complicated, which is able to analyze continuous girders crossed by a train of sprung and semi-sprung masses, and therefore can be used to treat the train-bridge interaction problem. As it is well known, the model using moving loads can be used for medium and long span bridges, while the interaction model is required for short bridges.

The mathematical bases of the different models have been developed by several authors (Biggs 1964, Fryba 1972). The behaviour of a simply supported beam subjected to a set of N moving loads travelling at a constant speed is described by the well-known partial differential equation

$$m\frac{\partial^2 y(x,t)}{\partial t^2}+EI\frac{\partial^4 y(x,t)}{\partial x^4}=\sum_{j=N_1}^{N_2}P_j\delta\left(x-V\left(t-t_j\right)\right) \qquad (1)$$

where:

$y(x,t)$ — vertical deflection of the beam at the point x and time t,
m — constant mass of the beam per unit length,
E — modulus of elasticity,
I — constant moment of inertia of the cross section of the beam,
N_1 — first axle load acting on the beam at time t,
N_2 — last axle load acting on the beam at time t,
P_j — the j-th axle load,
$\delta(x)$ — Dirac delta function,
V — constant speed of the set of loads (speed of the train),
t_j — time when the j-th axle load enters the bridge.

N_1 and N_2 are integer numbers varying with time as the train axles enter or leave the bridge (Figure 1). The damping of the beam has not been considered and will be introduced at a later stage.

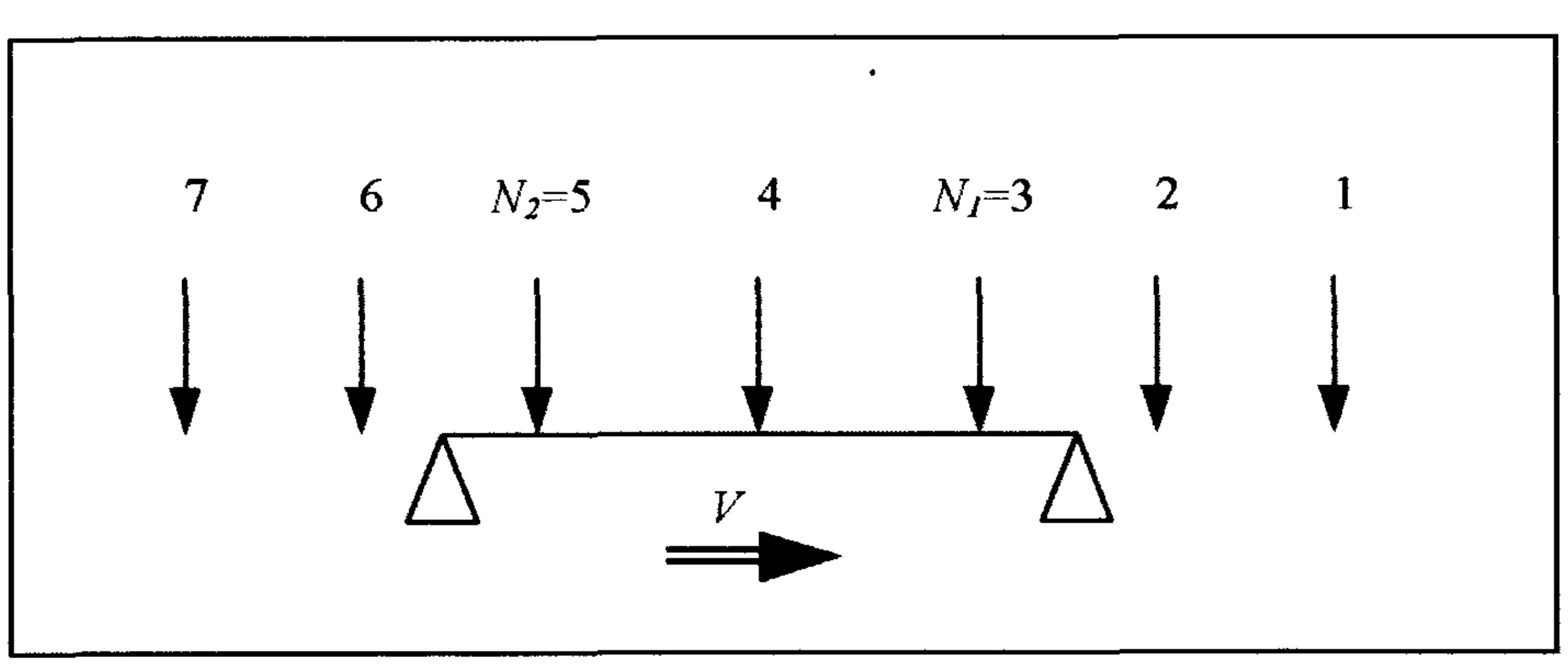


Figure 1. Set of moving loads at speed V.

The vertical deflection can be expressed as a combination of orthogonal modes by means of the usual family of sines

$$y(x,t)=\sum_{k=1}^{M}x_k(t)\sin\frac{k\pi x}{L} \qquad (2)$$

where M modes have been considered, L is the span of the beam and $x_k(t)$ is the time-varying contribution of the k-th mode to the deflection of the beam.

Using the approximation (2) in equation (1), multiplying by $\sin(n\pi x/L)$ and integrating over the length of the beam one obtains, after integrating by parts twice the second term in (1), the differential equation for $x_n(t)$:

$$\ddot{x}_n(t)+2\zeta_n\omega_n\dot{x}_n(t)+\omega_n^2x_n(t)=\frac{2}{mL}\sum_{j=N_1}^{N_2}P_j\sin\frac{n\pi V\left(t-t_j\right)}{L} \qquad (3)$$

where ω_n is the frequency of the n-th mode (rad/s) and a viscous damping ratio ζ_n has been introduced.

The forcing term, which is a function of time, changes its expression every time one of the loads enters or leaves the bridge, since this implies a change in N_1 or N_2 —or even in both simultaneously if one axle enters the bridge at the same time another one is leaving.

In order to overcome this initial difficulty, equation (3) is usually solved by means of numerical integration schemes, such as Runge-Kutta or Newmark-β methods. In our program a simple method for computing the exact values of $x_n(t)$ is used that avoids the main disadvantage of numerical procedures, i.e. the selection of an adequate time step for integration.

As shown before, the expression of the forcing term changes as the train moves forward and the pattern of loads acting on the beam is modified, i.e. the forcing term depends on the values of N_1 and N_2. Such being the case, the semiinfinite time interval $t\in[0, \infty)$ is divided in subintervals, each of them characterised by constant values of N_1 and N_2. The forcing term becomes a pure sinusoid within any of the subintervals, and the analytic solution can be computed provided the initial conditions are known.

As an example a set of four moving loads at an equidistant distance of 10m is considered. The set of loads is supposed to cross a 15m bridge at a constant speed V=20 m/s. Time is set to zero when the first load enters the bridge. The semiinfinite time interval $t\in[0, \infty)$ is then divided in eight subintervals as shown in Table 1.

Table 1. Subintervals of constant N_1 and N_2.

t(s)	0	0.5	0.75	1	1.25	1.5	1.75	2.25→ ∞
N_1	1	1	2	2	3	3	4	0
N_2	1	2	2	3	3	4	4	0

Within any of the subintervals the solution to equation (3) can be written as follows

$$x_n(t)=A\sin\frac{n\pi V}{L}t+B\cos\frac{n\pi V}{L}t+e^{-\zeta_n\omega_n t}\left(C\sin\omega_{dn}t+D\cos\omega_{dn}t\right) \qquad (4)$$

ω_{dn} being the frequency of the damped oscillation.

Coefficients A and B of the forced solution are easily computed for every subinterval since the forcing term is always known. Coefficients C and D can be obtained for the first subinterval from the homogenous initial conditions. Once A, B, C and D for the first subinterval are known, the initial conditions for the second subinterval can be evaluated (and therefore coefficients C and D) since they coincide with the final values of the first one. Performing the same operations for the rest of subintervals the analytic solution for the n-th mode is computed, and the total deflection of the bridge evaluated as a superposition of modal contributions.

On the other hand, the interaction model uses two-dimensional beam elements in order to represent the behaviour of the bridge, and a set of concentrated masses, springs and dampers to account for the characteristics of the train. The integration is carried out by means of a modified Newmark-β method. The mass, damping and stiffness matrix of the whole system are updated at every time step and the displacements of the axles and interaction forces interpolated using cubic hermitian polynomials. The time step for integration is chosen in such a way that it takes no less than 100 steps for any of the train axles to cross the shortest span of the bridge. The use of a longer time step proves inaccurate for the computation of the maximum displacements and accelerations.

3 IMPACT COEFFICIENT AND ACCELERATION BOUNDS

3.1 *Impact coefficient*

One of the most interesting ideas proposed by ERRI is the development of similarity formulae for different bridges of the same span.

According to ERRI (1997a), if the model with moving loads is assumed, it is possible to compute the maximum mid-span deflection and acceleration (f, a_{max}) of one bridge provided the values corresponding to another bridge of the same span L are known (f', a_{max}'). Say that m and n_0 are the mass per unit length and first natural frequency (Hz) of the first bridge, and m' and n_0' the values corresponding to the second bridge. If a constant damping ratio ζ is assumed for all modes the similarity formulae state that

$$f\left(L,\zeta,m,n_0,\frac{V}{n_0}\right)= \frac{m'}{m}\left(\frac{n_0'}{n_0}\right)^2 f'\left(L,\zeta,m',n_0',\frac{V'}{n_0'}\right) \tag{5.a}$$

$$a_{max}\left(L,\zeta,m,n_0,\frac{V}{n_0}\right)= \frac{m'}{m}a_{max}'\left(L,\zeta,m',n_0',\frac{V'}{n_0'}\right) \tag{5.b}$$

It should be emphasized that the wavelength λ must have the same value for both bridges:

$$\lambda=\frac{V}{n_0}=\frac{V'}{n_0'}=\lambda' \tag{6}$$

Relation (5.a) is also valid in the static case and therefore, denoting as f_{LM71} the static deflection due to the Load Model 71 proposed by the Eurocode-1, the impact coefficient can be computed as

$$\Phi=\frac{f\left(L,\zeta,m,n_0,\frac{V}{n_0}\right)}{f_{LM71}}=\frac{f'\left(L,\zeta,m',n_0',\frac{V'}{n_0'}\right)}{f'_{LM71}} \tag{7}$$

Relation (7) shows that the impact coefficient computed for two bridges of the same span length and damping has the same value provided the wavelengths λ and λ' are equal to each other.

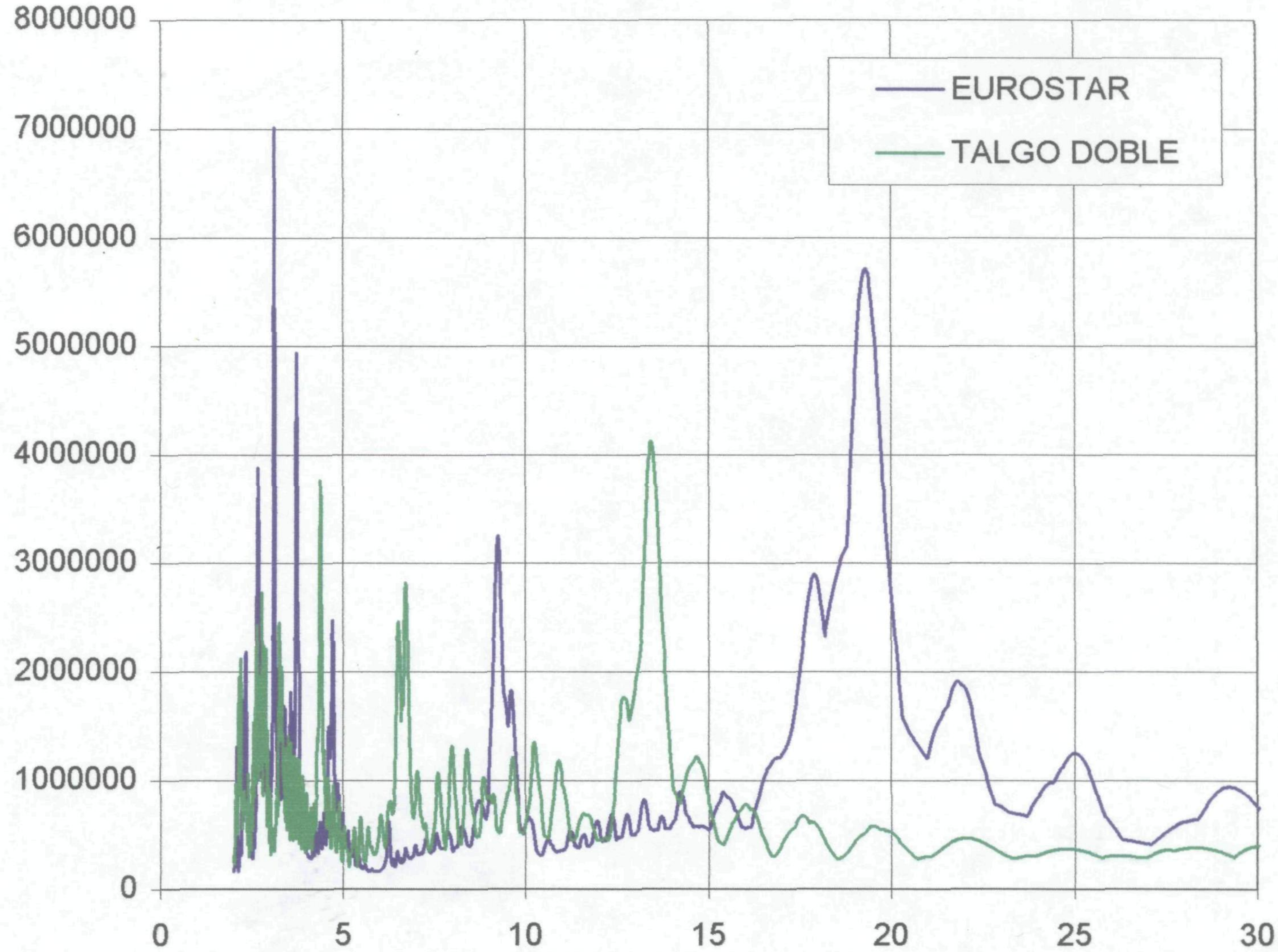
Based on the similarity formulae (5, 7), a dynamic analysis has been performed considering several simply supported bridges of different span length: L= 5, 7.5, 10, 15, 20, 25, 30, 35 and 40m. In this section the results concerning the impact coefficient are reviewed.

Five European high-speed trains have been studied, including the Spanish TALGO, the French THALY'S, the German ICE-2, the Italian ETR-Y and the EUROSTAR.

The maximum displacement has been evaluated for different values of speed ranging from 100 to 400 km/h, and the results have been plotted as a function of the wavelength $\lambda=V/n_0$. A damping ratio ζ=0.01 has been assumed. The displacements have been divided by the static deflection due to the Load Model 71, thus obtaining graphics of the impact coefficient Φ. Figure 2 shows the graphics for L=10m and L=20m.

For the values of the wavelength corresponding to the characteristic distances between the axles the highest amplifications occur: 18.7m for Thaly's and Eurostar, 26.4m for the Ice-2 and 26.1m for the Etr-Y. The Talgo train has a characteristic distance of 13.1m and lighter axle loads, which results in very low dynamic amplifications for span lengths greater than 20m. On the contrary, the amplifications observed in Figure 2 for shorter span lengths are of considerable importance.

This phenomenon of resonance vibration (Fryba and Naprstek 1998) can result in passenger


EUROSTAR
TALGO DOBLE
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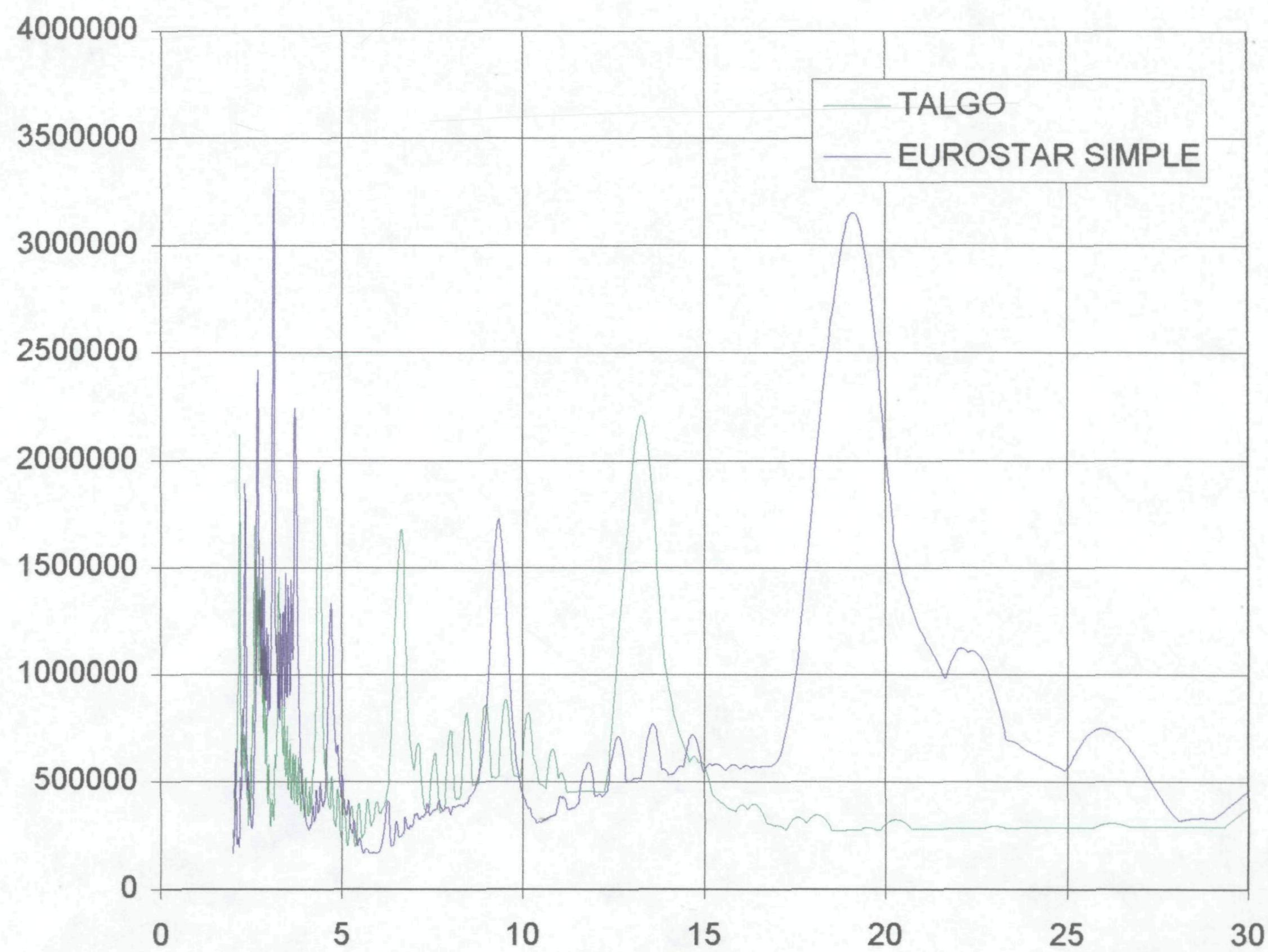
TALGO
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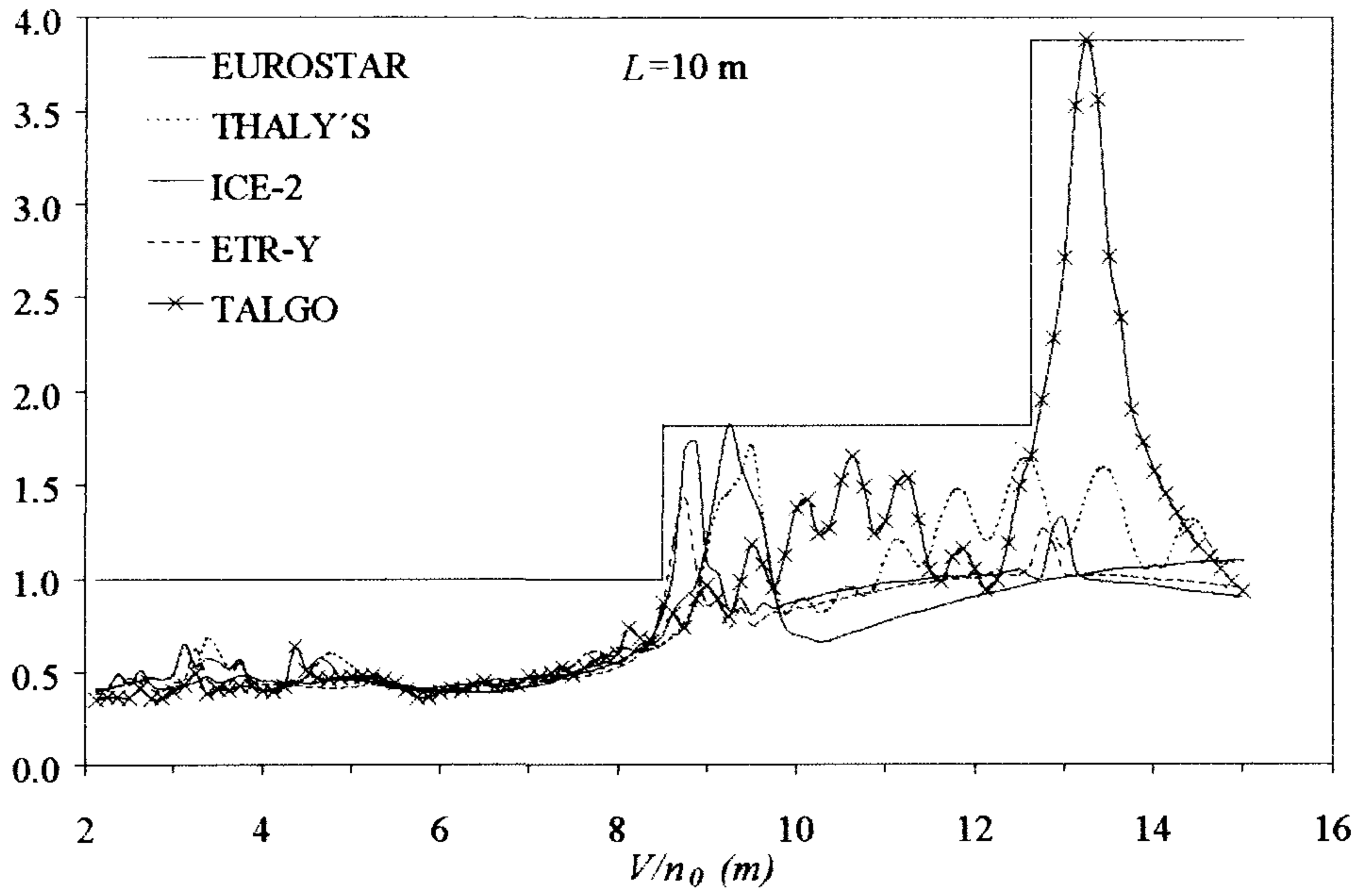

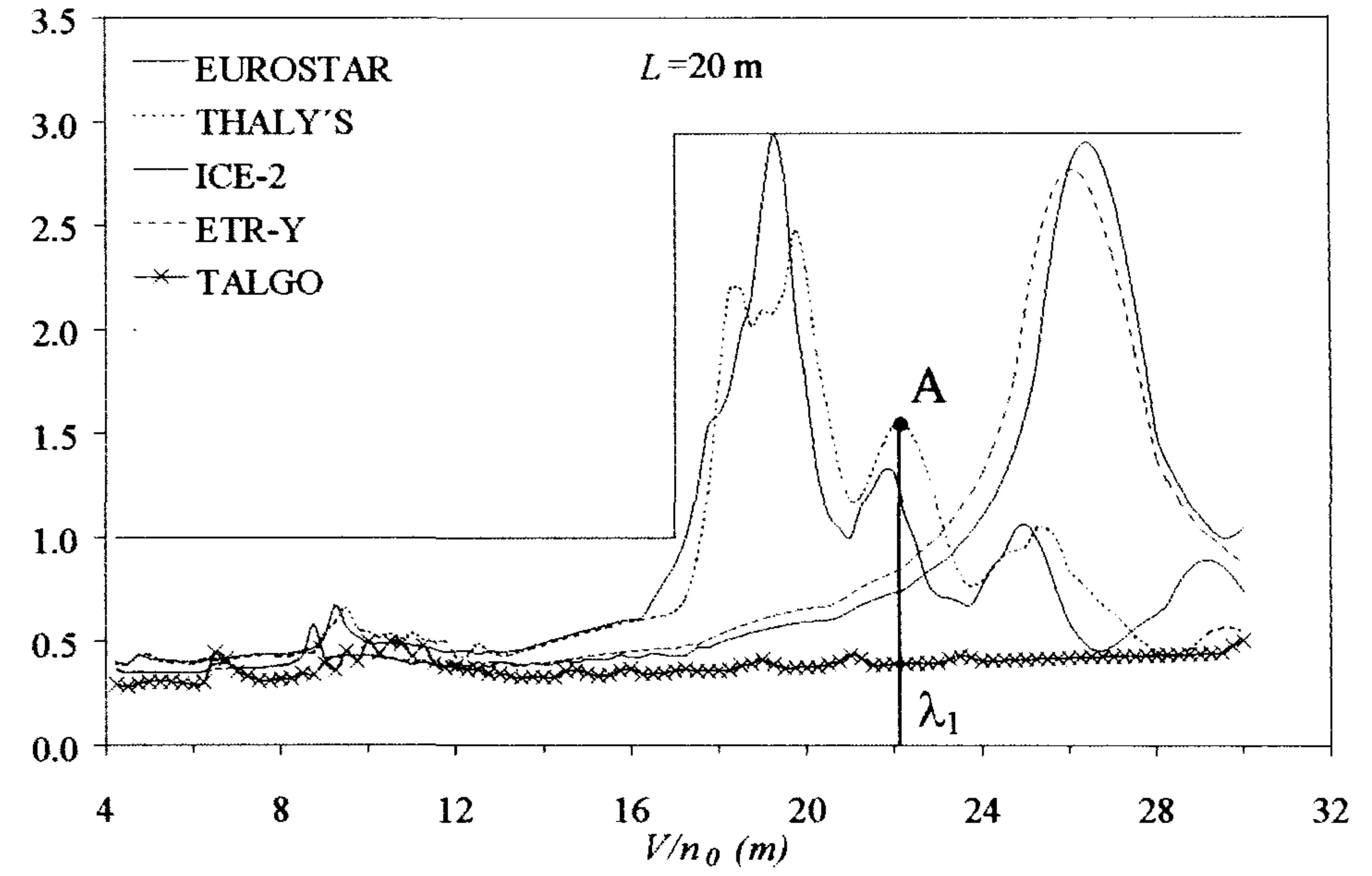

Figure 2. Impact coefficient Φ and envelope line for spans L=10m and L=20m. Five European high-speed trains considered.

discomfort, danger of derailment and serious damage to ballast if not properly accounted for.

For a given design speed of the line V_l and natural frequency of the bridge n_0, the impact coefficient Φ can be obtained from Figure 2 (point A). There is the possibility, however, of train travelling at a lower speed leading to a greater value of Φ and, therefore, the impact coefficient corresponding to $\lambda_1=V_l/n_0$ must be taken as the maximum of the values of Φ for all $\lambda<\lambda_1$.

This is accounted for by means of a staggered envelope line as shown in Figure 2. The values of the envelope line are always greater than one since it is intended to represent the dynamic amplification with respect to the static deflection due to Load Model 71.

If the design speed V_l has a given value the envelope of the impact coefficient can be plotted as a function of the first natural frequency, as shown in Figure 3 for L=20m and V_l=350 km/h.

The values of the envelope of the impact coefficient are summarised in Table 2 for several span lengths and design speed V_l=350 km/h. Similar tables are easy to produce for different values of V_l and L.

3.2 *Acceleration bounds*

Similar developments can be done for the accelerations. In this case the idea currently considered is to apply formula (5.b) and thus compute the value of the mass that has to be given to the bridge in order to respect the limit established by Eurocode (CEN 1995). If this is fixed to 5 m/s^2 and if all trains are considered again, it is possible to produce figures like Figure 4 that can be summarized as shown in Table 3. As can be seen, the masses are out of the usual range for short span bridges. As discussed below the experiments show that in those cases the assumption of one per cent damping is unrealistic.

3.3 *Train-bridge interaction*

ERRI (1997b) has pointed out that for short spans there is a strong interaction between bridge and train and that part of the vibration energy is literally moved out by the train. This phenomenon is accounted for by ERRI introducing a certain amount of additional damping in a simple model with moving loads.

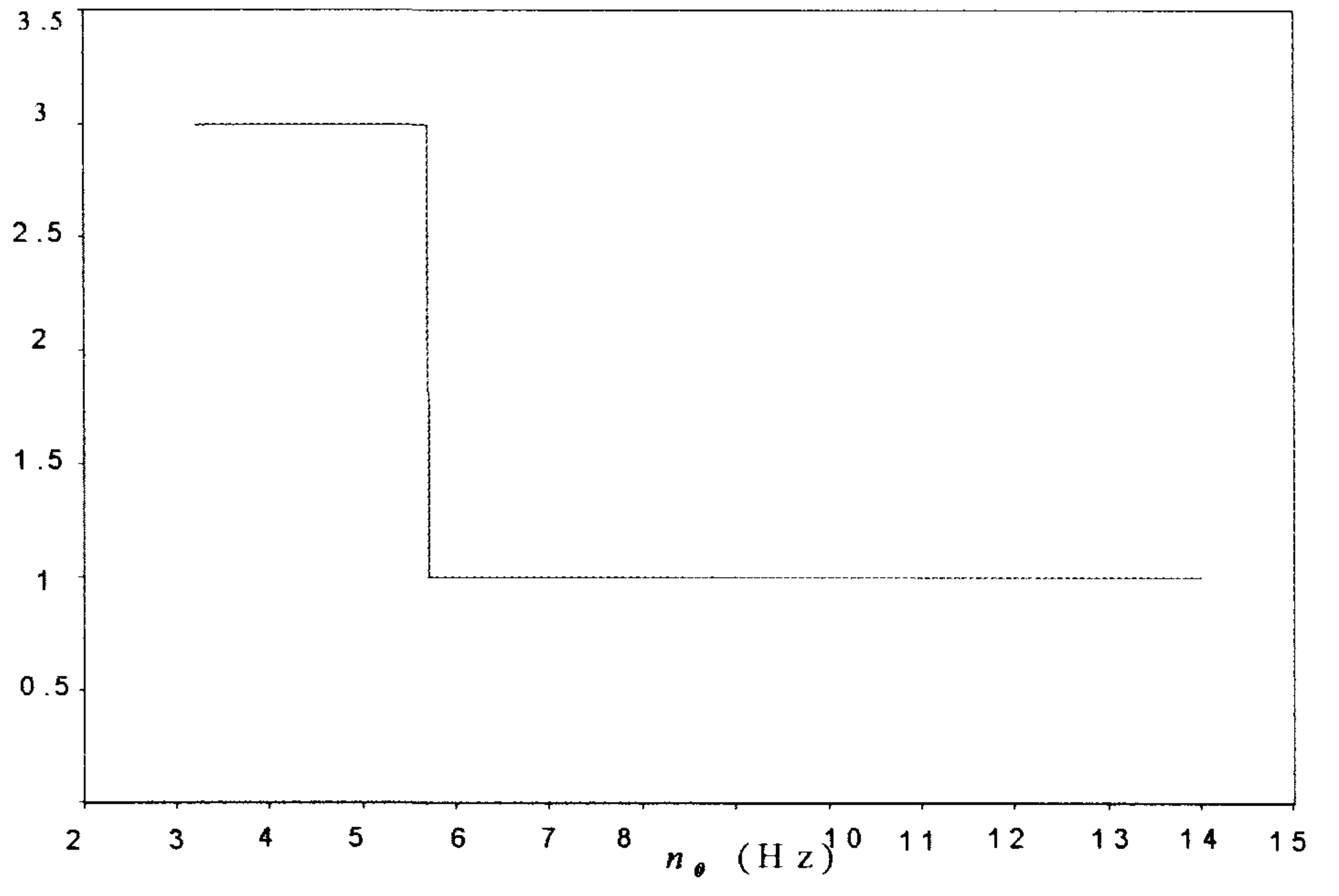

Figure 3. Envelope of the impact coefficient Φ as a function of frequency (span L=20m, design speed V_l=350 km/h).

On the other hand it is well known since long ago (Inglis 1934) that there are other focuses of damping related to the boundary restrictions exerted by the rails on the transition over the abutments.

Both ideas can help the reduction of the needed masses but the amount of additional damping that can be equivalent to those effects can not be easily fixed.

Figure 5 shows the impact coefficient computed for a 15m bridge crossed by ICE-2 using the simple model with moving loads and the interaction model. As can be seen, the values of the impact coefficient —or the maximum dynamic displacements—

obtained with the interaction model are lower than those obtained with the moving loads model. Moreover, this reduction of the impact coefficient, if evaluated as a percentage, is not the same for all the speed values.

Table 2. Impact coefficient as a function of span and frequency for design speed V_1=350 km/h.

Span	Frecuency (Hz)		
5 m	$16 \leq n_0 \leq 22{,}87$	$22{,}87 < n_0 \leq 26{,}36$	$26{,}36 < n_0 \leq 28{,}43$
	2,07	1,4	1,17
7.5 m	$10{,}67 \leq n_0 \leq 11{,}44$	$11{,}44 < n_0 \leq 15{,}35$	$15{,}35 < n_0 \leq 21$
	3,13	1,61	1,07
10 m	$8 \leq n_0 \leq 11{,}43$		$11{,}43 < n_0 \leq 16{,}93$
	1,82		1
15 m	$5{,}33 \leq n_0 \leq 5{,}65$	$5{,}65 < n_0 \leq 7{,}72$	$7{,}72 < n_0 \leq 12{,}5$
	4,49	1,89	1
20 m	$4 \leq n_0 \leq 5{,}72$		$5{,}72 < n_0 \leq 10{,}08$
	2,94		1
25 m	$3{,}5 \leq n_0 \leq 3{,}97$	$3{,}97 < n_0 \leq 5{,}25$	$5{,}25 < n_0 \leq 8{,}53$
	2,21	1,31	1
30 m	$3{,}15 \leq n_0 \leq 3{,}94$		$3{,}94 < n_0 \leq 7{,}44$
	1,49		1

In addition, the percentage of reduction also depends on the stiffness and frequency —or mass— of the bridge. Figure 6 shows the reduction obtained for thirty bridges of 15m with masses 6, 9, 12, 15 and 18 t/m, and frequencies ranging between the lower and upper limits recommended by UIC (Fryba 1996). The reduction has been evaluated for a value of speed that depends on the frequency of the bridge: the speed has been chosen in order to have a wavelength $\lambda = V/n_0$=8.8 because there is a resonance corresponding to this value (higher peak in Figure 5). The results are plotted as a function of (L/f_{LMF1}) for different values of frequency. It can be seen that for the lower values of frequency the percentage of reduction depends strongly on the frequency as well as on the stiffness. On the contrary, for values of frequency greater than 7.12 Hz the reduction depends mainly on the stiffness.

Figures 5 and 6 show that the reduction of dynamic effects obtained when considering the train-bridge interaction depends on the speed of the train as well as on the stiffness and frequency of the bridge. Therefore it is not easy to find a simplified expression for the additional damping that has to be introduced in the moving loads model in order to account for this phenomenon characteristic of short span bridges. Presently further analyses are being performed to find a satisfactory solution to this problem.

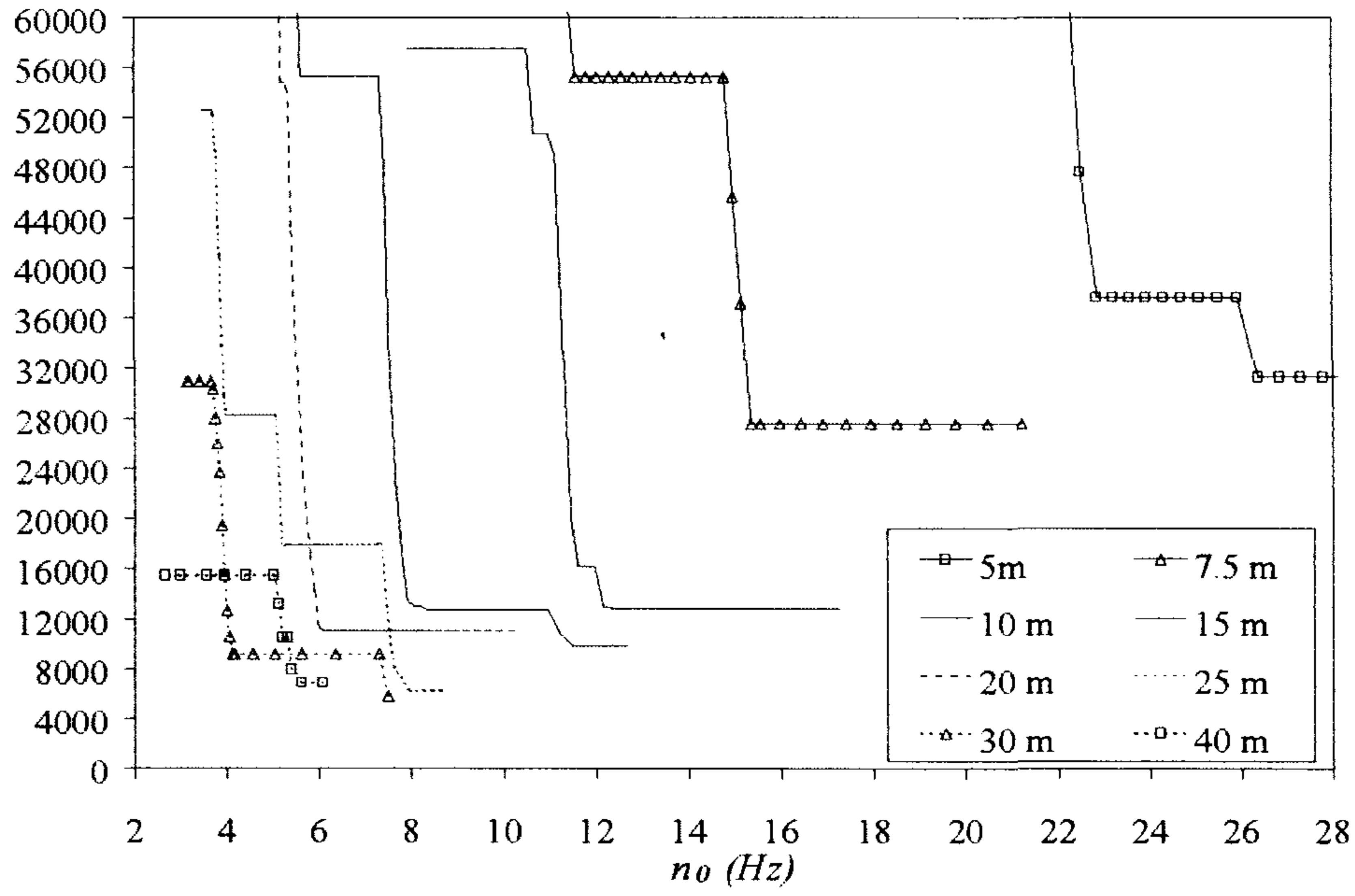

Figure 4. Minimum mass (kg/m) for design speed V_1=350km/h.

Table 3. Minimum mass for design speed V_1=350 km/h.

Span	Frecuency (Hz)	Minimum mass (kg/m)
5 m	$26{,}36 < n_0 \leq 28{,}43$	31500
	$22{,}87 < n_0 \leq 26{,}36$	38000
	$16 \leq n_0 \leq 22{,}88$	68000
7.5 m	$15{,}35 < n_0 \leq 21$	28000
	$11{,}44 < n_0 \leq 15{,}35$	55500
	$10{,}67 \leq n_0 \leq 11{,}44$	111500
10 m	$12{,}15 < n_0 \leq 16{,}93$	13000
	$10{,}65 < n_0 \leq 12{,}15$	51000
	$8 \leq n_0 \leq 10{,}65$	57500
15 m	$11{,}3 < n_0 \leq 12{,}15$	10500
	$7{,}97 < n_0 \leq 11{,}3$	14000
	$5{,}58 < n_0 \leq 7{,}97$	56500
	$5{,}33 \leq n_0 \leq 5{,}58$	106500
20 m	$5{,}98 < n_0 \leq 10{,}08$	11500
	$5{,}18 < n_0 \leq 5{,}98$	55000
	$4 \leq n_0 \leq 5{,}18$	78500
25 m	$7{,}94 < n_0 \leq 8{,}53$	6500
	$5{,}18 < n_0 \leq 7{,}94$	18000
	$3{,}97 < n_0 \leq 5{,}18$	28500
	$3{,}5 \leq n_0 \leq 3{,}97$	53000
30 m	$4{,}1 < n_0 \leq 7{,}44$	9500
	$3{,}15 \leq n_0 \leq 4{,}1$	31000
35 m	$5{,}95 < n_0 \leq 6{,}63$	4500
	$5{,}2 < n_0 \leq 5{,}95$	15000
	$2{,}87 \leq n_0 \leq 5{,}2$	18000
40 m	$5{,}5 < n_0 \leq 6$	7000
	$2{,}66 \leq n_0 \leq 5{,}5$	15500

4 CONCLUSIONS

1. Following the similarity formulae (5, 6) proposed by ERRI, the impact coefficient and maximum accelerations have been evaluated for several bridges of span lengths ranging from five to forty meters. The Spanish TALGO and four European high-speed trains have been considered.

2. The TALGO train, having a characteristic distance of 13.1m between axle sets and axle loads of some 170kN —except for the locomotive loads, which are heavier—, produces dynamic effects of

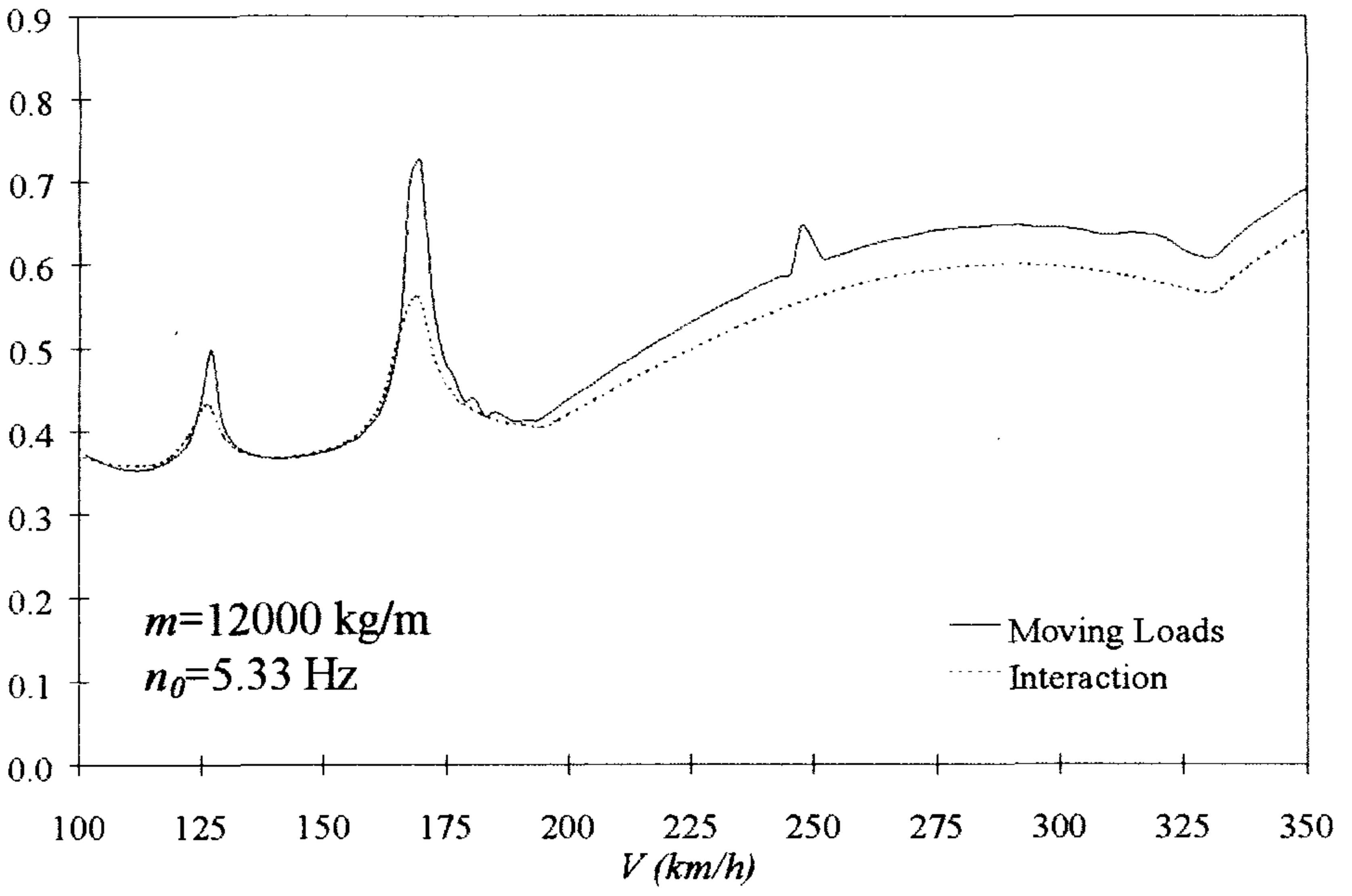

Figure 5. Comparison of the Moving Loads and Interaction models for a bridge of span L=15m: Impact coefficient Φ.

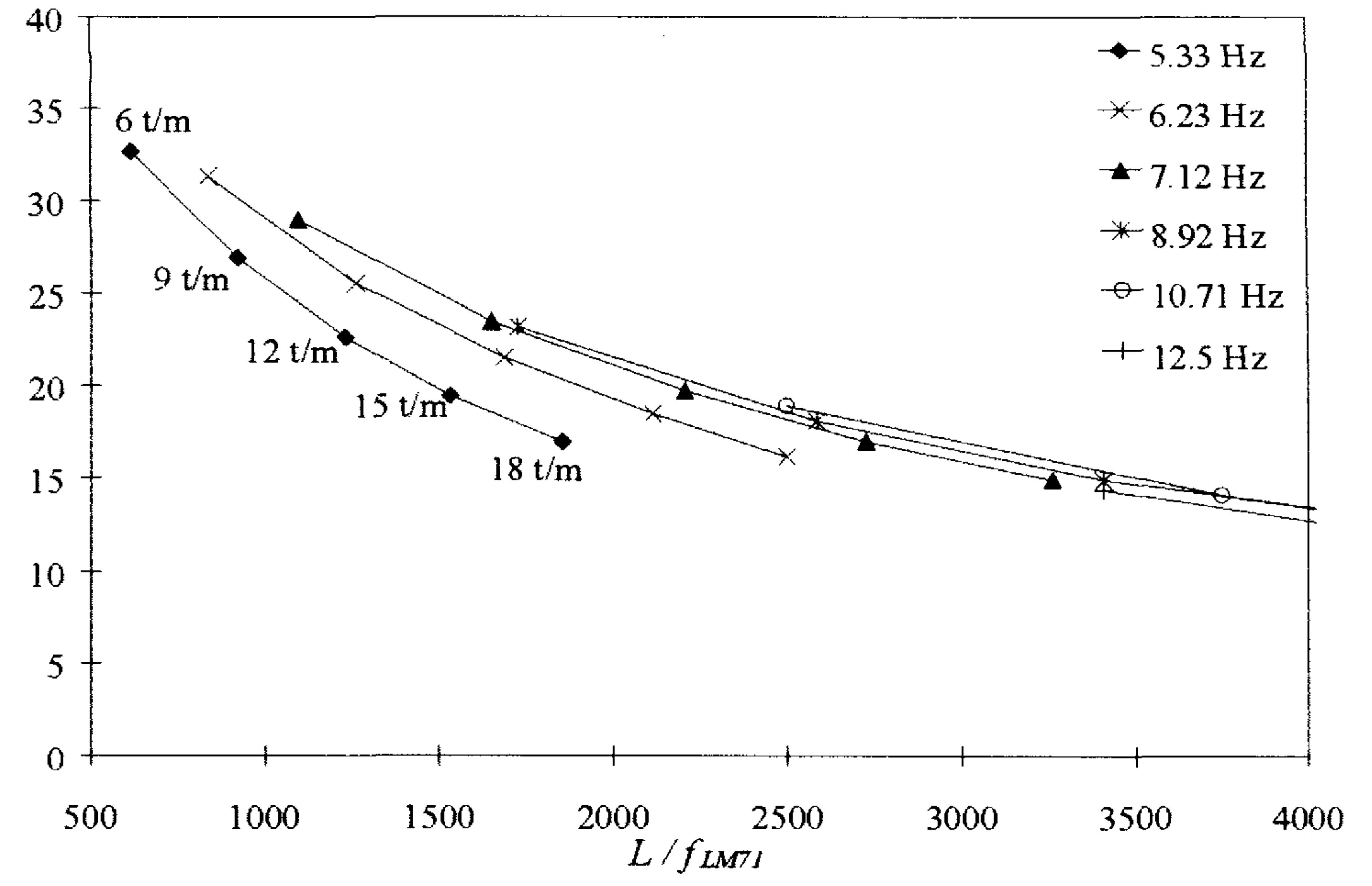

Figure 6. Reduction of the impact coefficient (%): comparison of the Moving Loads and Interaction models for L=15m.

considerable importance on bridges of span length shorter than 15m. For longer bridges the resonance phenomena caused by the other European trains result in much stronger amplifications than those observed for the TALGO.

3. The impact coefficient Φ plotted as a function of the wavelength can be simplified adopting an envelope line. Using such simplification the values of Φ are obtained, for a given design speed of the line, as a function of the span and first natural frequency of the bridge (Table 2).

4. Similarly, using relation (6) a minimum mass can be computed that is required in order to satisfy the limitation of the accelerations. The values of the minimum mass can be summarized as in Table 3 for a given design speed. As can be seen, the values corresponding to short bridges are much greater than the usual ones.

5. The train-bridge interaction is a phenomenon that significantly reduces the dynamic effects in short span bridges ($L \leq 25$m approximately) when compared with the results obtained using simple models with moving loads. The amount of reduction depends on the speed as well as on the frequency and stiffness of the bridge. Therefore it is not possible at this moment to give a simple rule to account for this reduction and further research is needed in order to find a satisfactory solution.

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